THE CONTINUITY CORRECTION IN THE NORMAL APPROXIMATION

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There arise experimental situations in educational research where a binomial model is assumed and inferential statistics based upon normal curve theory are employed to test hypotheses (e.g., % test for the mean). One example is student guessing behavior on multiple choice tests (See for example, Ebel,) where the probability of obtaining a correct answer by guessing is 1. K = number of choices per item.

As is well known, the use of the normal curve to evaluate probability levels of Bernoulli sequences, or more specifically cumulative Bernoulli sequences, depends upon the asymptotic convergence of the binomial distribution function to that of the normal distribution function (e. g., Fisz, 1963, or almost any standard text on mathematical statistics or probability). What is also well appreciated is that for finite samples, the utilization of normal integrals to evaluate binomial situations involves some error. Many studies have investigated and catalogued facets of the error related to such variables as sample size and population proportion, among others. (See, for example, Mosteller, 1961; Jeffreys, 1961; Raff, 1956; Mood and Graybill, 1963; Dayton, 1964; Plackett, 1964; Munroe, 1951.)

One of the more widely used techniques for making the normal integral more valid for binomial experiments is the Yates-type continuity correction. Typically, a constant of one-half is subtracted from the absolute value of the difference between the sample value and the assumed population mean in forming the Z statistic.

Thus, instead of the analytically derived $Z = \frac{X - np}{(npq)\frac{1}{2}}$, we have $Z_{C} = |\frac{X - np}{(npq)\frac{1}{2}}$.

This "correction" factor is usually attributed to Yates and dates to 1921 in course outlines of the Department of Applied Statistics at University College, London (Plackett, 1964).

It is the contention of the present paper that the purpose and operation of the Yates-type correction in the normal approximation to the binomial is still misunderstood, particularly among educational statisticians. If one samples the standard texts in educational research and statistics (e.g., Fryer, 1966; Cornell, 1956; Walker and Lev, 1953), one comes away with the fact that the purpose of the correction for continuity is to effect better fitting normal curves for the binomial model.

Todhunter (1931) reports Yates as first applying that connotation to integrals of $f(\mathbb{Z}c)$ compared to $f(\mathbb{Z})$. Lindgreen (1964) employs an intuitive approach to demonstrate that using $\mathbb{Z}c$ provides less error in the approximation than using \mathbb{Z} . Richardson (1944) concurs in this interpretation, while Cochran (1953) and Dayton (1964) devote papers to finding other subtractors than one-half to provide better fitting normal approximations.

In recent years, several authors, (e.g., Montel and Greenhouse, 1968) have demonstrated, by using computer-based procedures, that the continuity correction may not provide better fitting theoretical curves at all, but will tend to reduce the percent of false rejections of the null hypothesis.

The present paper investigated this relationship of using the normal approximation to test binomial hypotheses but considering an additional factor of importance in educational research which has not been explored previously by applied statisticians.

Typically, the research setting in which the binomial model is assumed (e.g., test-taking behavior) calls for the formulation of the nondirectional hypothesis. This concept is related to the removing of <u>a-priori</u> bias between two opposing methods. It is the contention of some educational statisticians (e.g., Raths, 1964) that research hypotheses in educational experiments should almost always be non directional.

In the binomial situation, if we are given N and \mathcal{H} and wish to calculate $P(X \leq X_0)$, where Xo is a given number of defined successful outcomes in a Bernoulli sequence, a non-directional hypothesis would be defined as follows:

Directional probability refers to the probability of all successes at least as extreme from np as Xo and in the same algebraic direction from np as X. Non-directional probability includes the directional total plus the probability of all successes at least as extreme from np as (X^1) and in the same algebraic direction from np as (X^1) , where (X^1) is the nearest integer value to the X^1 whose Z score is the additive inverse of the Z score of X.

Thus, non-directional hypotheses yield probabilities of sample occurrence greater than or equal to directionally calculated probabilities. Very few studies of the continuity correction and the error in using normal integrals have considered the case of non-directional binomial hypothesis. Those that have (e.g., Mosteller, 1961), have assumed that symmetry would negate studying the inverse sample results. However, the binomial distribution is not always symmetrical about its mean value, and one cannot assume the error function will simply be proportioned on both sides np.

Thus, the present paper investigated the relative error distribution in using the standard unit normal curve for binomial sequences. Computer generated samples were employed to calculate the relative error for directional and non-directional hypotheses and for corrected and uncorrected Z-statistics. Since the skewness of the binomial distribution appears in the literature to be of paramount importance in assessing the degree of convergence between the normal and binomial, the present study controlled for skewness.

For a given value of skewness, binomial probabilities were calculated for various n and p combinations. It can be shown (Fisz, 1963) that for the binomial $SK = \frac{q}{(npq)^2}$

Since skewness is not bounded and since the number of usable samples generated varies with the skewness, and further, since practical study had shown few instances where SK>1, one was chosen as an upper bound for SK. The number of usable samples for study is limited by SK, since we can show that $p = \frac{1}{2}(1 + (\frac{ny^2}{l_4 + ny^2})^{\frac{1}{2}})$, y = SK

Thus, for large values of SK, p will be very small and the associated Z statistic will be too large for meaningful comparisons. The skewness was incremented by 0.02 from zero through one, and the program generated the relative error distribution, summary statistics and the percent of rejections of null hypotheses by the normal approximation when the binomial probabilities were greater than .05 or .01, respectively. This procedure was replicated for the directional and non-directional hypotheses and uncorrected and corrected Z statistics.

There were little data to support the belief, held in educational research, that Zc unqualifyingly produces significantly better fitting normal curves to the binomial model. On the other hand, the Zc's were associated with somewhat lower mean relative errors, but not for all values of skewness.

In fact, for directional hypotheses, small binomial skewness resulted in better fitting curves for uncorrected than for corrected Z statistics. As the skewness increased Zc did yield normal curves with less relative error. It is interesting to note that the corrective factor of Zc was typically greater for non-directional than for directional hypotheses in lowering the mean value of the relative errors, and that as the skewness increased Zc's corrective power also increased. Further, even the uncorrected non-directional curves had less relative error than the directional counterparts. The following table lists the mean relative errors for selected values of binomial skewness.

Moreover, the distributions of relative error were all positively skewed and platykurtic. As the binomial skewness increased, the relative error distributions became less positively skewed, a result of the general tendency of the error to increase as the binomial skewness increased. The non-directional errors were the most positively skewed. The distributions of relative error were non-normal in form. The second characteristic of the present investigation dealt with the percent of Type I errors under the four modes of the Z statistic. The data presented show clearly that employing the Yates-type correction factor results in a substantial reduction of Type I errors in using normal integrals to evaluate binomial hypotheses over the uncorrected Z statistics. This is an expected result for directional hypotheses, but is not an obvious result for the non-directional case. In fact, as the following table shows, the reduction in Type I errors is the largest for non-directional corrected hypotheses. Additionally, for some values of binomial skewness, there were no Type I errors made when using the nondirectional Zc.

Of interest is to note that, while in general the corrected Z statistics had less percent of Type I errors than the uncorrected Z scores, such percent for the most part increases as the binomial skewness increases. At the .01 level, the percent of Type I errors is generally larger than at the .05 levels across all four modes of Z statistics.

Further, the disparity between the percent of Type I decision errors for corrected and uncorrected Z statistics is not as great at the .01 level than at the .05 levels.

It is also important to note that even when the binomial skewness is zero and the directional and non-directional error distributions are identical, the percent of Type I errors are unequal. This again emphasizes the need to investigate the non-directional case for binomial hypotheses.

The present paper investigated the distributions of relative error in using normal integrals to assess probabilities for binomial hypotheses under four modes of the standardized Z statistic: directional uncorrected, directional corrected, non-directional uncorrected, non-directional corrected. Computer based procedures generated relative error distributions, summary statistics and Type I error tables for each mode of Z statistic while controlling for the binomial skewness. Skewness was incremented from 0.00 to 1.00 by steps of 0.02, and n and p combinations for each skewness value, n = 2(100)(2).

In summary, then, the following conclusions seem tenable:

- 1. In general, when normal integrals are used for binomial hypotheses, the mean relative errors increase as the binomial skewness increases.
- 2. Non-directional Z statistics yield better fitting normal curves relative to the binomial probabilities than do directional Z statistics.

- 3. In general, corrected Z statistics are associated with somewhat less mean relative errors than uncorrected Z statistics. However, for binomial skewness near zero, directional uncorrected Z's have less mean error than directional corrected Z's.
- 4. The percent of Type I decision errors made when normal probabilities are used generally increases as the binomial skewness increases.
- 5. Corrected Z's are associated with lower percents of Type I errors than uncorrected Z's.
- 6. Non-directional Z's are associated with lower percents of Type I errors than directional Z's.
- 7. The percent of Type I decision errors made using normal integrals is generally inversely related to the alpha level of the null hypothesis.

It would appear that when using normal integrals to evaluate binomial hypotheses, the skewness of the binomial affects both the relative error in the calculation of probabilities and the percent of error in rejecting the null hypothesis. The Yates-type continuity correction combined with a non-directional hypothesis minimizes both the relative error and the error in decision compared to the other modes.

It would seem that for nearly symmetrical binomials (SK<.10), a "better fit" is obtained with the directional uncorrected than the directional corrected. However, even in this case, one lessens the percent of Type I decision errors by using Zc.

Thus, the data of the present study seem to support the contention that the primary effect of the continuity correction in the normal approximation to the binomial is to provide a more conservative test of hypothesis rather than to obtain better fitting normal integrals.

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Table 1. Mean Relative Errors

Binomial Skewness	Directional Uncorrected	Directional Corrected	Non-Directional Uncorrected	Non-Directional Corrected	
0.00	.235	.095	.235	.095	
.02	1.232	2.115	.291	.118	
.04	2.367	5.623	.325	.132	
.06	1.577	3.876	.338	.141	
•08	.688	1.267	.352	.147	
.10	.489	·659	.362	.164	
.12	.446	. ليلبل	. 374	.172	
.14	•434	· 342	. 381	.191	
.16	.455	• 306	• 391	.199	
.18	.463	.285	• 399	.214	
.20	.465	.277	.408	.226	
•22	.464	.278	.411	.223	
.24	.474	. 314	.421	.235	
•26	.481	.301	.421	.249	
• 30	.497	.292	.428	.255	
•34	.522	• 300	•447	.271	
• 38	•538	•299	․ կկ7	.314	
.42	• 550	.300	.468	• 306	
•46	•553	•289	.464	•295	
•50	•570	• 305	•473	.363	
.60	•587	.312	•499	.282	
.70	.600	. 318	•473	• 305	
.80	.618	• 337	•523	• 390	
.90	.636	• 372	•550	• 363	
1.00	•602	• 327	.478	• 304	

Table 2. Percent of Type I Decision Errors at .05 and .01 Levels

Binomial Skewness	Direc	Directional Uncorrected		tional ctional	Non-Dire Uncorr	Non-Directional Uncorrected		Non-Directional Corrected	
	.05	.01	.05	.01	.05	.01	.05	.01	
0.00	3.10	2.96	0.00	0.00	3.70	2.81	0.00	0.00	
0.02	1.08	3.06	0.53	0.98	3.82	3.65	0.27	0.35	
.04	3,56	3.95	0.61	0.91	1.02	3,92	0.30	0.30	
.06	5.00	L.88	1.03	2.18	L.13	L.25	0.34	0.34	
.08	5.80	5.56	1.03	2.32	5.20	1.84	0,39	0.51	
.10	6.72	6.06	1.13	2.82	6.16	1.61	0.12	0.70	
.12	6.40	6.68	1.07	3.83	4.13	5.83	0.31	0.00	
.14	7.00	6.70	1.48	3.95	5.78	ĥ.11	0.16	0.00	
.16	7.90	7.58	1.58	4.74	3.39	4.19	0.18	0.00	
.13	7.82	7.82	0.74	4.28	6.29	3.74	1.49	0.18	
.20	7.13	8.74	1.76	4.31	6.24	4.46	0.00	0.59	
.22	6.38	9.19	1.24	5.59	6.19	5.63	0.00	0.62	
.24	8.72	10.08	1.72	5.36	2.13	5.43	0.00	1.72	
.26	8.32	9.74	3.39	4.97	3.04	5.43	0.00	0.68	
• 30	13.97	11.35	2.20	4.96	2.18	5.68	0.00	2.45	
.40	15.08	10.42	3.49	4.74	3.55	6.21	0.00	0.75	
.50	10.27	10.96	3.09	8.77	3.20	8.68	5.41	1.29	
.60	6.43	12.14	1.08	8.38	1.43	8.33	0.00	2.71	
•70	6.81	14.99	0.63	7.57	1.64	8.45	0.00	5.99	
•80	11.32	15.60	11.19	14.08	10.40	12.84	0.00	5.42	
•90 ·	16.67	17.01	1.26	5.04	2.43	16.67	0.00	1.26	
1.00	17.89	20.33	6.12	21.43	17.75	18.29	0.00	20.92	